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Renormalization group and decoupling in curved space 3. The case of spontaneous symmetry breaking

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Abstract: We continue investigation of the renormalization group and decoupling of the quantized massive fields in curved space [1]. In the present work we analyze the vacuum sector of a theory, where fields gain their masses due to the Spontaneous Symmetry Breaking (SSB), the case providing the remarkable exception from the Appelquist-Carazzone (AC) theorem in the matter fields sector. Due to the non-minimal coupling between the scalar field and gravity, the vacuum expectation value (VEV) of the scalar is not constant and the induced gravitational action includes an infinite number of non-local terms. Despite this property the theory is renormalizable and the low-energy decoupling in the higher-derivative gravitational sector performs similar to the AC theorem.

Keywords: Renormalization Group, Spontaneous Symmetry Breaking, Physics of the Early Universe.

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Contents

1. Introduction

The renormalization group and decoupling of quantized massive fields [2] in curved space are important (but maybe not always noticed) aspects of many modern theories of quantum gravity. For example, the effective low-energy quantum gravity [3, 4] is based on the assumption that the decoupling really takes place. This enables one to separate the quantum effects of the heavy fields from the ones of the light fields and in particular of gravitons. The decoupling is in the heart of the cosmological applications [5, 6] of the semi-classical approach to gravity (see, e.g. [7, 8]), where the metric is considered as a classical external background for the quantized matter fields. In a more general framework, the low-energy spectrum of the (super)string theory includes large amount of massive degrees of freedom and the consistency of the low-energy predictions of string theory implies that the virtual loops of these excitations do not affect the gravity dynamics for the sole reason they have a large mass and decouple. The standard Appelquist-Carazzone — like [2] form of decoupling of the quantized massive fields in curved background has been always anticipated [3, 4, 9], however the practical calculations of the decoupling have been started only recently by the authors in [1]. In these papers we have performed the calculations of the 1-loop Feynman diagrams for the graviton propagator on the flat background using a physical mass-dependent renormalization scheme, and also equivalent covariant calculations in the second order in the curvature tensor. In the higher derivative vacuum sector we met an expected form of decoupling, similar to the one of the Appelquist-Carazzone theorem in a matter sector. Unfortunately, for the cosmological Λ and inverse Newton $1/G$ constants one does not see the decoupling and even the β-functions themselves. The most probable reason is the restricted power of the available calculational methods, which are essentially based on the perturbative expansion on the flat background or on the equivalent covariant procedure. Indeed, we have to assume that the β-functions for Λ and $1/G$ exist, for otherwise we would meet a disagreement between the mass-dependent renormalization scheme and the completely covariant minimal subtraction $\overline{MS}$ scheme where these two β-functions can be easily obtained [8]. It is worth noticing that the simplest assumption concerning the form of decoupling for the cosmological constant leads to the potentially testable running of the cosmological constant in the late Universe [10] and therefore this problem deserves a special attention.

All the results of [1] concern the decoupling of the massive fields in curved space-time. But there is an interesting aspect of the decoupling which has to be considered separately. In many cases the fields which are massless in the initial classical lagrangian become massive due to the SSB mechanism and we need to know whether the decoupling takes place, also, for these fields. The special interest to this problem is due to the well-known fact that in the matter field sector the theories with the SSB violate the Appelquist-Carazzone theorem (see, e.g. [11]). One can, naturally, wonder whether something like that happens or not in the case of gravity. Indeed all massive particles in the Standard Model gain their masses due to the SSB, one can guess that this is also the case for the Grand Unification Theories. Furthermore, the possibility of the supersymmetry breaking due to the SSB can not be completely ruled out [12], and therefore the understanding of decoupling in an external gravitational field in the theories with SSB looks rather general problem.

The purpose of the present article is to clarify the issue of decoupling in the SSB theories. The paper is organized as follows. The section 2 is devoted to the description of the SSB at the tree level in the case of the non-minimal coupling of the scalar (Higgs) field with gravity. We find out that the low-energy induced gravitational action includes, along with the Einstein-Hilbert term, an infinite set of the non-local terms with the same dimension. In section 3 the general discussion of the one-loop corrections to the induced gravitational action in the theory with SSB and for the case of the minimal interaction is given. In particular, the gauge-fixing invariance of the quantum corrections in the theory with SSB is proved explicitly. In sections 4 and 5 we generalize these considerations to the general situation with the non-minimal interaction. In section 4 the logarithmic divergences are calculated and the general scheme of renormalization of the vacuum sector in the theory with SSB is outlined. In particular, we show that the non-local terms must be included into the vacuum action and that these terms must be renormalized at quantum level. In section 5 the physical mass-dependent scheme of renormalization is applied to the SSB theory with the general non-minimal coupling between scalar and gravity. An explicit form of the β-functions in the vacuum sector are derived and the low energy decoupling (analog of the Appelquist and Carazzone theorem) is established for the parameters corresponding to all higher derivative terms. Finally, we summarize the results in section 6.

2. SSB and the induced vacuum action

In this paper we shall deal with the following classical action of charged scalar φ coupled to the abelian gauge vector A_μ:

$$S=\int d^4x\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+g^{\mu\nu}(\partial_\mu-ieA_\mu)\varphi^*(\partial_\nu+ieA_\mu)\varphi+\mu_0^2\varphi^*\varphi-\lambda(\varphi^*\varphi)^2+\xi R\varphi^*\varphi\right\}. \tag{2.1}$$

The generalization for the non-abelian theory would be straightforward, but there is no reason to consider it because we are interested in the one-loop vacuum effects and the results are indeed the same for both abelian and non-abelian cases.

Our first purpose is to investigate the SSB at the classical level. The VEV for the scalar field is defined as a solution of the equation

$$-\Box v + \mu_0^2\, v + \xi R\, v - 2\lambda v^3 = 0\,. \tag{2.2}$$

If the interaction between scalar and metric is minimal $\xi = 0$, the SSB is standard and simple, because the vacuum solution of the last equation is constant

$$v_0^2 = \frac{\mu_0^2}{2\,\lambda}\,. \tag{2.3}$$

In the last expression we have introduced a special notation v_0 for the case of a minimal interaction, in order to distinguish it from the solution v of the general equation (2.2). Starting from (2.3), the conventional scheme of the SSB and the Higgs mechanism does not require serious modifications because of the presence of an external metric field. However, the consistency of the quantum field theory in curved space requires the non-minimal interaction such that $\xi \neq 0$ (see, e.g. [8] for the introduction). For the general case of the non-constant scalar curvature one meets, instead of eq. (2.3), another solution $v(x) \neq$ const. Hence, we can not ignore the derivatives of v and, unfortunately, the solution for the VEV can not be obtained in a closed and simple form.

Our main interest in this paper will be the decoupling of massive fields at low energies, when the values of scalar curvature are small. Therefore, we can try to consider (2.3) as the zero-order approximation and find the solution of the eq. (2.2) in the form of the power series in the curvature tensor (or equivalently in ξ)

$$v(x) = v_0 + v_1(x) + v_2(x)\, + \cdots\,. \tag{2.4}$$

For the first order term $v_1(x)$ we have the following equation

$$-\Box v_1 + \mu^2 v_1 + \xi R\, v_0 - 6\lambda v_0^2\, v_1 = 0\,, \tag{2.5}$$

and the solution has the form

$$v_1 = \frac{\xi\, v_0}{\Box - \mu^2 + 6\lambda v_0^2}\, R = \frac{\xi\, v_0}{\Box + 4\lambda v_0^2}\, R\,, \tag{2.6}$$

where we used (2.3).[1] In a similar way, we find

$$v_2 = \frac{\xi^2\, v_0}{\Box + 4\lambda v_0^2}\, R\, \frac{1}{\Box + 4\lambda v_0^2}\, R - \frac{6\,\lambda\, \xi^2\, v_0^3}{\Box + 4\lambda v_0^2}\, \left(\frac{1}{\Box + 4\,\lambda v_0^2}\, R\right)^2\,, \tag{2.7}$$

where the operator in each parenthesis acts only on the curvature inside this parenthesis.

[1] The Green's functions introduced here are defined as follows. Since the consequent quantum calculations will be performed in the euclidean space, we prefer to impose the boundary conditions which follow from the analytic continuation to the Minkowski space of the corresponding euclidean expression.

Contrary to that, the left operator $[\Box + 4\lambda v_0^2]^{-1}$ in the first term at the *r.h.s.* of the last equation acts on all expression to the right of it. In general, here and below the parenthesis restrict the action of the differential or inverse differential (like $[\Box + 4\lambda v_0^2]^{-1}$) operators.

Of course, one can continue the expansion of v to any desirable order. If we replace the SSB solution $v(x)$ back into the scalar section of the action (2.1) we obtain the following result for the induced low-energy action of vacuum:

$$S_{\rm ind} = \int d^4x\sqrt{-g}\,\{g^{\mu\nu}\,\partial_\mu v\,\partial_\nu v + (\mu_0^2 + \xi R)\,v^2 - \lambda\,v^4\}\,. \tag{2.8}$$

It is remarkable that, instead of the conventional cosmological constant and Einstein-Hilbert term, here we meet an infinite series of non-local expressions due to non-locality of (2.4). Making an expansion in the powers of the curvature tensor, in the second order we obtain

$$\begin{aligned} S_{\rm ind} = \int d^4x\sqrt{-g}\,\Big\{ &- v_1\Box v_1 + \mu^2\,(v_0^2 + 2v_0v_1 + 2v_0v_2 + v_1^2) \\ &- \lambda\,(v_0^4 + 4v_0^3v_1 + 4v_0^3v_2 + 6v_0^2v_1^2) + \xi R\,(v_0^2 + 2v_0v_1)\Big\} + \mathcal{O}(R^3)\,. \end{aligned} \tag{2.9}$$

Now, using the equation (2.5), after a small algebra we arrive at the following action of induced gravity

$$S_{\rm ind} = \int d^4x\sqrt{-g}\,\Big\{\lambda v_0^4 + \xi R v_0^2 + \xi^2\,v_0^2\,R\,\frac{1}{\Box + 4\,\lambda v_0^2}\,R + \cdots\Big\}\,. \tag{2.10}$$

The first term here is the induced cosmological constant, which is supposed to almost cancel with its vacuum counterpart (see, e.g. the discussion in [5]). The second term is a usual induced Einstein-Hilbert action, which also has to be summed up with the corresponding vacuum term. Formally, both observables: the cosmological and the Einstein-Hilbert terms, are given by the sums of the vacuum and induced contributions. However, there is a great difference between the two terms from physical point of view. The observable cosmological constant is extremely small compared to the magnitude of the λv_0^4, e.g. in the Standard Model of particle physics. Hence there is an extremely precise cancelation between the vacuum and induced cosmological constants (see, e.g. [13] for the introduction to the cosmological constant problem and also [5] for the discussion of the possible quantum effects). At the same time, the situation for the Einstein-Hilbert term is quite different. The overall coefficient here is nothing but the inverse Newton constant $1/16\pi G = M_P^2/16\pi$, where $M_P \approx 10^{19}$ GeV is a Planck mass. Of course, the magnitude of this quantity is huge compared to the induced term. The exception is indeed possible if we assume a SSB phenomenon at the Planck scale, but at the lower energies one can not distinguish the difference between this "induced" gravitational action and the "original" vacuum one. Therefore, at low energies the local induced Einstein-Hilbert term is irrelevant compared to the classical (vacuum) gravitational action.[2]

[2]The remarkable exception is the possibility to have totally induced Einstein-Hilbert term (see, e.g. [14] and references therein). Recently, the model of induced gravity found interesting applications in the black hole physics [15].

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Besides the usual local terms, the induced tree-level gravitational action includes an infinite set of the non-local terms due to the non-constant VEV of the scalar field. These terms are somehow similar to the nonlocalities which have been recently discussed in [16] in relation to the higher derivative theories and the cosmological constant problem. The appearance of the non-local terms in the induced action (2.10) is remarkable, also, for other reasons. Although the coefficients of these terms are very small compared to the vacuum Einstein-Hilbert term, the non-localities do not mix with the local terms and, in principle, can lead to some physical effects. If considering the low energy SSB phenomena in the framework of the SM, the non-local terms are irrelevant at low energies due to the large value of the mass. But, if we assume that there is an extremely light scalar (e.g. quintessence), whose mass is of the order of the Hubble parameter and which has a potential admitting a SSB, then the non-localities may become relevant and in particular lead to observable consequences. In the next sections we will not discuss these issues and will instead concentrate on the quantum one-loop corrections to the induced action (2.10).

The vacuum expectation value of the scalar field is not constant also in the case of a theory with solitons. Although the problem of calculating quantum corrections in a theory with solitons (for a review, see, e.g. [17]) is mathematically very similar, physically it is rather different because in this case the occurence of non-constant vacuum solutions is the consequence of the proper dynamics of the theory (namely, the existence of nonequivalent vacua and topologically stable vacuum solutions interpolating between them) rather than the effect of an external agent like a non-constant background gravitational field in our case. The form of the coordinate dependence in the theory with solitons is a fixed property of the theory and cannot be changed by adjusting the external parameters. In this respect, theories with non-constant background fields have more freedom.

3. The minimal interaction case and gauge fixing independence

In the previous section, we considered the SSB in curved space-time at the classical level. The next problem is to derive the quantum corrections to the vacuum action from the theory (2.1) with the SSB. The contributions of the free massive scalar, fermion and vector fields to the effective action of vacuum were already calculated in [1]. Here we shall generalize the same approach for the case when the masses emerge as a result of the SSB in curved space-time.

Let us start from the simplest case of the minimal interaction $\xi = 0$. The effective action $\Gamma[\varphi, g_{\mu\nu}]$ of the scalar field can be presented as the perturbative expansion

$$\Gamma[\varphi, g_{\mu\nu}] = S_{cl}[\varphi, g_{\mu\nu}] + \hbar\,\bar{\Gamma}^{(1)}[\varphi, g_{\mu\nu}] + \mathcal{O}(\hbar^2)\,. \tag{3.1}$$

In this paper we restrict our attention by the one-loop order and therefore consider only the $\bar{\Gamma}^{(1)}[\varphi, g_{\mu\nu}]$ term. Then the effective equation for the VEV of scalar field has the form

$$\frac{\delta S_{cl}}{\delta\varphi} + \hbar\,\frac{\delta\bar{\Gamma}^{(1)}}{\delta\varphi} = 0\,. \tag{3.2}$$

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Since S_{cl} is given by the scalar sector of (2.1), the equation (3.2) can be rewritten as

$$-\Box\varphi + \mu^2\varphi - 2\lambda(\varphi^*\varphi)\varphi + \hbar\,\frac{\delta\bar{\Gamma}^{(1)}}{\delta\varphi} = 0\,. \tag{3.3}$$

Let us now present the scalar field as $\varphi = v + \hbar\phi$, where v is the solution of the classical equation $\delta S_{cl}/\delta\varphi = 0$ and $\hbar\phi$ is a quantum correction. Then we find, in the first order in $\hbar$, the following relation:

$$-\Box\phi + \mu^2\phi - 6\lambda v^2\phi + \hbar\,\frac{\delta\bar{\Gamma}^{(1)}[v,g]}{\delta v} = 0\,. \tag{3.4}$$

The expansion in $\hbar$ yields

$$\begin{aligned}\Gamma[\varphi, g_{\mu\nu}] &= S_{\rm cl}[v+\hbar\phi,\, g_{\mu\nu}] + \hbar\,\bar{\Gamma}^{(1)}[v+\phi,\, g_{\mu\nu}] + \cdots \\ &= S_{\rm cl}[v,\, g_{\mu\nu}] + \hbar\phi\,\frac{\delta S_{cl}[v,g]}{\delta v} + \hbar\,\bar{\Gamma}^{(1)}[v,\, g_{\mu\nu}] + \mathcal{O}(\hbar^2)\,. \end{aligned} \tag{3.5}$$

Taking into account the equation of motion $\delta S_{\rm cl}(v,g)/\delta v = 0$, we arrive at the useful formula

$$\Gamma[v+\hbar\phi,\, g_{\mu\nu}] = S_{\rm cl}[v,\, g_{\mu\nu}] + \hbar\,\bar{\Gamma}^{(1)}[v,\, g_{\mu\nu}] + \mathcal{O}(\hbar^2)\,. \tag{3.6}$$

The last relation holds even for the non-minimal scalar field, and we shall use it in what follows. The equation (3.6) shows that at the one-loop level one can derive the effective action as a functional of the classical VEV. In the minimally interacting theory this VEV is just a constant, but in the general non-minimal case the classical VEV itself is a complicated expression (2.4).

Consider the SSB in the theory (2.1) with $\xi = 0$. For this end we define $\varphi = v + h + i\eta$ and replace it back to the action. As far as we are interested in the one-loop effects, we can keep the terms of the second order in the quantum fields h, η and disregard higher order terms. In this way we arrive at the expression for the quadratic in quantum fields part of the action

$$S^{(2)} = \int d^4x\sqrt{-g}\left\{(\partial_\mu h)^2 + (\partial_\mu\eta)^2 - \frac{1}{4}F_{\mu\nu}^2 + 2evA_\mu\nabla^\mu\eta + e^2v^2A_\mu A^\mu - 4\lambda v^2h^2\right\}, \tag{3.7}$$

where we used notation $(\partial h)^2 = g^{\mu\nu}\partial_\mu h\partial_\nu h$. Let us introduce the 'tHooft gauge fixing condition, depending on an arbitrary parameter α

$$S_{GF} = -\,\frac{1}{2\alpha}\int d^4x\sqrt{-g}\,(\nabla_\mu A^\mu - 2\,\alpha\,ev\,\eta)^2\,. \tag{3.8}$$

The expression for the action with the gauge fixing term is

$$\begin{aligned} S^{(2)} + S_{GF} = \int d^4x\sqrt{-g}\Big\{ &-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\alpha}\,(\partial_\mu A^\mu)^2 + e^2v^2A_\mu A^\mu + \\ &+ (\partial_\mu h)^2 + (\partial_\mu\eta)^2 \;-\; 4\lambda v^2h^2 - 2\,\alpha e^2v^2\eta^2\Big\} + \cdots\,, \end{aligned} \tag{3.9}$$

where we kept only the terms of the second order in the quantum fields A^μ, h, η.

The action of the Faddeev-Popov gauge ghosts can be obtained in a standard way as

$$S_{GH} = \int d^4x\sqrt{-g}\,\bar{C}\,\left(\Box + 2\,\alpha e^2 v^2\right) C\,. \tag{3.10}$$

After all, from the equations (3.9) and (3.10) we find that the one-loop corrections to the vacuum effective action are given by the contributions of the fields A_μ, h, η, $\bar{C}$ and C

$$\begin{aligned}\bar{\Gamma}^{(1)}[\varphi, g_{\mu\nu}] = \frac{i}{2}\,\mathrm{Tr}\ln\left[\delta^\mu_\nu\,\Box - \Big(1-\frac{1}{\alpha}\Big)\nabla^\mu\nabla_\nu - R^\mu_\nu + 2e^2v^2\,\delta^\mu_\nu\right] + \\ + \frac{i}{2}\,\mathrm{Tr}\ln(\Box + 4\lambda v^2) + \frac{i}{2}\,\mathrm{Tr}\ln(\Box + 2\alpha\, e^2v^2) - i\,\mathrm{Tr}\ln(\Box + 2\,\alpha e^2 v^2)\,.\end{aligned} \tag{3.11}$$

In the general case of an arbitrary α the first term of the last expression is related to the functional determinant of a non-minimal massive vector field. This kind of operator have been never elaborated in the literature,[3] moreover for the particular value $\alpha = 0$ the expression (3.11) includes the contributions of several massless modes, jeopardizing the expected low-energy decoupling. For all other values of α all the degrees of freedom in (3.11) are massive. Furthermore, in the particular case $\alpha = 1$ the above expressions have only the well-known contributions of the minimal massive vector and scalars. Indeed, the β-functions for both these cases were calculated in [1], where the universality of the decoupling of the massive fields at low energies has been demonstrated. Thus, the decoupling is guaranteed if we can prove the gauge-fixing independence of the effective action (3.11).

Let us remember that there are general theorems concerning the on-shell gauge fixing independence. This dependence in the effective action is possible but for the one-loop divergences it is always proportional the classical equations of motion. As far as we are investigating the vacuum sector of the theory and the equations of motion involve matter fields, the gauge fixing dependence can not take place. Furthermore, according to the considerations at the beginning of this section, at one loop order the extremum (up to the higher order corrections in the Planck constant) is equivalent to the classical extremum. Thus, the mentioned theorems should be directly applicable in our case. However, since the gauge fixing independence has special importance here, it is worth verifying it explicitly, at least for the particular case $\xi = 0$. The methods for investigating the gauge fixing dependence have been developed in [18], and in this section we shall apply the modified version of these methods for the case of (3.11). As far as the gauge-fixing independence is established, the derivation of the vacuum β-functions in the theory with SSB can be easily performed using eq. (3.11) and the results of [1]. The explicit calculation will be postponed for the next section, where we shall consider a more general theory with an arbitrary ξ.

Following [18], one can evaluate the difference between the euclidean one-loop correction with an arbitrary value of the gauge parameter α and the same correction with $\alpha = 1$.

$$\bar{\Gamma}^{(1)}[\varphi, g_{\mu\nu};\,\alpha] - \bar{\Gamma}^{(1)}[\varphi, g_{\mu\nu};\,1]\,. \tag{3.12}$$

[3]The reader can easily evaluate the contribution of this operator to the UV divergences using the consideration of the rest of this section.

The first term in eq. (3.11) has the form

$$\hat{\mathcal{F}}(\alpha) = \mathcal{F}_\mu^\nu(\alpha) = \delta_\mu^\nu \Box - \left(1 - \frac{1}{\alpha}\right) \nabla_\mu \nabla^\nu - R_\mu^\nu + m^2\, \delta_\mu^\nu\,, \tag{3.13}$$

where we denoted $m^2 = 2e^2 v^2$. Consider the difference

$$-\frac{1}{2}\,\mathrm{Tr}\ln \hat{\mathcal{F}}(\alpha) + \frac{1}{2}\,\mathrm{Tr}\ln \hat{\mathcal{F}}(1) = -\frac{1}{2}\,\mathrm{Tr}\ln\left[\delta_\mu^\nu - \left(1 - \frac{1}{\alpha}\right)\nabla_\mu \nabla^\nu\, \frac{1}{\Box + m^2 - R_{\dots}}\right]. \tag{3.14}$$

Let us use the identity, derived in [18] for an arbitrary vector field A_ν

$$\left(\nabla^\mu\, \frac{1}{\Box - R_{\dots}} - \frac{1}{\Box}\,\nabla^\mu\right) A_\mu = 0\,. \tag{3.15}$$

The calculation of gauge fixing dependence performed in [18] concerns the massless gauge field, and in our case we have a massive field. That is why we need to generalize the identity (3.15) for the massive case

$$\left(\nabla^\mu\, \frac{1}{\Box + m^2 - R_{\dots}} - \frac{1}{\Box + m^2}\,\nabla^\mu\right) A_\mu = 0\,. \tag{3.16}$$

This generalization can be performed by expanding (3.16) into the series in m^2. First we present the propagator for the massive case as a series

$$\frac{1}{\Box + m^2 - R_{\dots}} = \sum_{n=0}^{\infty}(-1)^n\, \frac{(m^2)^n}{(\Box - R_{\dots})^{n+1}}\,. \tag{3.17}$$

The zero-order part of (3.17) is the original identity (3.15). In the first order in m^2 we need to prove the identity

$$\left[\nabla^\mu\, \frac{1}{(\Box - R_{\dots})^2} - \frac{1}{\Box^2}\,\nabla^\mu\right] A_\mu = 0\,. \tag{3.18}$$

This can be easily done by presenting it in the form

$$\left(\nabla^\mu\, \frac{1}{\Box - R_{\dots}} - \frac{1}{\Box}\,\nabla^\mu\right)\frac{1}{\Box - R_{\dots}}\, A_\mu + \frac{1}{\Box}\left(\nabla^\mu \frac{1}{\Box - R_{\dots}} - \frac{1}{\Box}\,\nabla^\mu\right) A_\mu = 0\,, \tag{3.19}$$

where we have used the fact that the vector $A'_\mu = (\Box - R_{..})^{-1}\, A_\mu$ also satisfies the identity (3.15). The same operation can be applied at any order in m^2, therefore we proved the identity (3.16).

Using this identity, one can rewrite the difference (3.14) as

$$-\frac{1}{2}\,\mathrm{Tr}\ln\left[\delta_\mu^\nu - \left(1 - \frac{1}{\alpha}\right)\nabla_\mu \frac{1}{\Box + m^2}\nabla^\nu\right] == \frac{1}{2}\,\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \frac{1}{\alpha}\right)^n \left(\nabla_\mu \frac{1}{\Box + m^2}\nabla^\nu\right)^n. \tag{3.20}$$

Using the relation $\mathrm{Tr}(A \cdot B \cdot C) = \mathrm{tr}(C \cdot A \cdot B)$ and taking the trace over the indices μ and ν, after a small algebra, (3.20) can be transformed into the expression involving only the scalar operators (here and below we disregard the $\mathrm{Tr}\ln$ of constants which are not relevant, e.g. in the dimensional regularization)

$$-\frac{1}{2}\,\mathrm{Tr}\left[\ln\,\hat{\mathcal{F}}(\alpha)-\ln\,\hat{\mathcal{F}}(1)\right]=-\frac{1}{2}\,\mathrm{Tr}\ln\left(\frac{\Box+\alpha m^2}{\Box+m^2}\right). \tag{3.21}$$

The gauge dependent part of the contribution of the Higgs scalar is zero because the Higgs mass M_H does not depend on the parameter α. The gauge dependent part of the contribution of the Goldstone scalar (3.11) is exactly the same as the vector counterpart (3.21)

$$\frac{1}{2}\,\mathrm{Tr}\ln\left(\frac{\Box+2\alpha\,e^2v^2}{\Box+2\,e^2v^2}\right)=-\frac{1}{2}\,\mathrm{Tr}\ln\left(\frac{\Box+\alpha m^2}{\Box+m^2}\right). \tag{3.22}$$

Finally, the difference between the two ghost operators (3.8) contributes as

$$\mathrm{Tr}\ln\left(\frac{\Box+\alpha m^2}{\Box+m^2}\right). \tag{3.23}$$

In total, three contributions (3.21), (3.22) and (3.23) give zero, and therefore the one-loop part of the vacuum effective action in the theory with SSB is gauge fixing independent. As it was shown above, this also means that in the vacuum sector the quantum effects of the theory with the SSB satisfy the Appelquist and Carazzone theorem [2] and manifest the decoupling at low energies. In this respect the gravitational vacuum quantum effects are very different from the quantum effects in the matter sector where the mentioned theorem may be violated for the masses of the SSB origin (see, e.g. [11]).

4. $\overline{\text{MS}}$-scheme renormalization in the non-minimal case

Let us first derive the one-loop divergences in the theory with the SSB, using the background field method and the Schwinger-DeWitt technique [19]. Starting from the action (2.1), we are going to integrate over the matter fields φ, A_μ on the background of the classical metric $g_{\mu\nu}$. In the spontaneously broken phase the scalar field φ takes the VEV v corresponding to the solution of the equation (2.2). Indeed, this solution (2.4) depends exclusively on the metric. Therefore, we perform the background shift of the scalar variable according to

$$\varphi=v+h+i\eta\,, \tag{4.1}$$

where h and η are real scalar quantum fields (Higgs and Goldstone). Hence, we face a problem of deriving the divergences in the theory with quantum fields A_μ, h, η, while the background fields include metric and v, which, in turn, also depends on the metric. As we shall see in what follows, the renormalization of the theory looks rather standard in terms of $g_{\mu\nu}$ and v. However, it looks very unusual if we take the expression (2.4) for v into account and express the effective action in the terms of metric.

In order to derive the one-loop quantum correction one needs the part of the action which is bilinear in quantum fields. Elementary calculations give the following result for the sum of the action (2.1) and the gauge-fixing term (3.8):

$$S^{(2)}+S_{GF}=\int d^4x\sqrt{-g}\Big\{\frac{1}{2}\,A_\mu\Box A^\mu+\frac{1}{2}\left(1-\frac{1}{\alpha}\right)(\nabla_\mu A^\mu)^2-A^\mu A^\nu R_{\mu\nu}+\frac{1}{2}\,M_A^2A^2+ \\ +(\partial_\mu h)^2+(\partial_\mu\eta)^2-M_H^2h^2-M_\eta^2\eta^2-2\alpha e\eta\,A^\mu(\partial_\mu v)\Big\}\,, \tag{4.2}$$

where we introduced new notations

$$M_A^2 = 2e^2 v^2\,, \qquad M_h^2 = 6\lambda v^2 - \mu_0^2 - \xi R\,, \qquad M_\eta^2 = 2e^2 v^2 + 2\lambda v^2 - \mu_0^2 - \xi R\,. \tag{4.3}$$

One can rewrite these quantities in a more useful way. First we introduce

$$\xi \mathcal{K} = 2\lambda\left(v^2 - v_0^2\right) = 2\lambda v^2 - \mu_0^2\,. \tag{4.4}$$

After replacing (2.4), (2.6) and (2.7) into (4.4), in the lowest order in curvature we obtain

$$\xi \mathcal{K} = \frac{2\xi v_0^2}{\Box + 4\lambda v_0^2}\, R + \mathcal{O}(R^2)\,. \tag{4.5}$$

If we are interested in the low-energy effect, then the derivatives of curvature are very small compared to v_0^2. Then we can expand the Green function, in the expression above, as follows

$$\frac{1}{\Box + 4\lambda v_0^2} = \frac{1}{4\lambda v_0^2}\left(1 - \frac{\Box}{4\lambda v_0^2} + \cdots\right) + \mathcal{O}(\Box R)\,. \tag{4.6}$$

In the low-energy approximation we arrive at the representation

$$\xi \mathcal{K} = \xi R + \frac{\text{higher derivative terms}}{v_0^2}\,. \tag{4.7}$$

In what follows we will perform the quantum calculations for the complete expressions for the x-dependent VEV and (4.4), and not for the approximate low-energy one (4.7). For the general case the $(\Box v)/v$ term admits the following representation in terms of $\mathcal{K}$:

$$\frac{(\Box v)}{v} = \frac{\mu_0^2\, v + \xi R\, v - 2\lambda\, v^3}{v} = \xi R + 2\lambda v_0^2 - 2\lambda v^2 = \xi R - \xi \mathcal{K}\,. \tag{4.8}$$

In the new notations the elements of the expansion (4.3) may be written in the form

$$\begin{aligned}
M_A^2 &= m^2 + \frac{e^2}{\lambda}\,\xi\mathcal{K}\,, & m^2 &= 2e^2 v_0^2\,;\\
M_h^2 &= m_h^2 - \xi\, R + 3\,\xi\,\mathcal{K}\,, & m_h^2 &= 4\lambda v_0^2\,;\\
M_\eta^2 &= m^2 - \xi\, R + \left(\frac{e^2}{\lambda} + 1\right)\xi\mathcal{K}\,,
\end{aligned} \tag{4.9}$$

where m and m_h are the masses of the fields after SSB. Indeed, their values are the same as in the minimal $\xi = 0$ case.

Coming back to the calculation of divergencies, since the one-loop effective action is gauge-fixing invariant, (see the discussion in the previous section) we can put $\alpha = 1$. Then, making a change of variables

$$h = \frac{i}{\sqrt{2}}\,\tilde{h}\,, \qquad \eta = \frac{i}{\sqrt{2}}\,\tilde{\eta}\,,$$

we arrive at the following useful form of the bilinear part of the action (4.3):

$$S^{(2)} + S_{GF} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,\tilde{h}\,\hat{\mathcal{H}}_h\,\tilde{h} + \frac{1}{2}\,(A^\mu\,\tilde{\eta})\,\hat{\mathcal{H}}\,\begin{pmatrix} A_\nu \\ \tilde{\eta}\end{pmatrix}\right\}, \tag{4.10}$$

where the operators have the form

$$\hat{\mathcal{H}}_h = \Box + M_h^2 \qquad \text{and} \qquad \hat{\mathcal{H}} = \left\| \begin{array}{cc} \delta^\nu_\mu \Box - R^\nu_\mu + M_A^2 \delta^\nu_\mu & -i\sqrt{2}\, e\, (\partial_\mu v) \\ -i\sqrt{2}\, e\, (\nabla^\nu v) & \Box + M_\eta^2 \end{array} \right\| . \tag{4.11}$$

Both operators $\hat{\mathcal{H}}_h$ and $\hat{\mathcal{H}}$ have the standard structure $\hat{1}\Box + \hat{\Pi}$ and the algorithm for the divergences is well known (notice that the masses are included into the operators Π for all the fields)

$$\frac{i}{2}\,\mathrm{Tr}\ln(\hat{1}\Box + \hat{\Pi})\Big|_{\mathrm{div}} = \frac{1}{2(4\pi)^2(2-\omega)} \int d^4x\,\sqrt{-g}\,\mathrm{tr}\Big\{ \frac{\hat{1}}{180}\left(R^2_{\mu\nu\alpha\beta} - R^2_{\mu\nu} + \Box R\right) + \frac{1}{2}\,\hat{P}^2 + $$
$$+ \frac{1}{12}\hat{S}_{\mu\nu}\hat{S}^{\mu\nu} + \frac{1}{6}\,\Box \hat{P} \Big\} , \tag{4.12}$$

where ω is the parameter of dimensional regularization and

$$\hat{P} = \hat{\Pi} + \frac{\hat{1}}{6}\,R \qquad \text{and} \qquad \hat{S}_{\mu\nu} = [\nabla_\mu\,,\,\nabla_\nu]\,,$$

where the commutator of covariant derivatives is calculated in the corresponding vector space. Using the formula (4.12) we obtain

$$\frac{i}{2}\,\mathrm{Tr}\ln\hat{\mathcal{H}}_h\Big|_{\mathrm{div}} = \frac{1}{2(4\pi)^2(2-\omega)} \int d^4x\,\sqrt{-g}\,\,\mathrm{tr}\,\Big\{ \frac{\hat{1}}{180}\left(R^2_{\mu\nu\alpha\beta} - R^2_{\mu\nu} + \Box R\right) + \tag{4.13}$$
$$+ \frac{1}{2}\left(M_h^2 + \frac{1}{6}\,R\right)^2 + \frac{1}{6}\,\Box\left(M_h^2 + \frac{1}{6}\,R\right)\Big\}$$

for the contribution of the field h. In the second case the operators $\hat{1}$ and $\hat{\Pi}$ have matrix form

$$\hat{1} = \left\| \begin{array}{cc} \delta^\nu_\mu & 0 \\ 0 & 1 \end{array} \right\| \qquad \text{and} \qquad \hat{\Pi} = \left\| \begin{array}{cc} -R^\nu_\mu + M_A^2 \delta^\nu_\mu & -i\sqrt{2}\, e\, (\partial_\mu v) \\ -i\sqrt{2}\, e\, (\nabla^\nu v) & M_\eta^2 \end{array} \right\| . \tag{4.14}$$

Performing calculations according to (4.12), we arrive at

$$\frac{i}{2}\,\mathrm{Tr}\ln\hat{\mathcal{H}}\Big|_{\mathrm{div}} = \frac{1}{2(4\pi)^2(2-\omega)} \int d^4x\sqrt{-g}\,\times$$
$$\times\,\mathrm{tr}\Big\{ \frac{1}{36}\left(R^2_{\mu\nu\alpha\beta} - R^2_{\mu\nu}\right) + \frac{1}{2}\,R^2_{\mu\nu} - 2e^2(\nabla v)^2 + 2\left(M_A^2 + \frac{1}{6}\,R\right)^2 -$$
$$- \left(M_A^2 + \frac{1}{6}\,R\right)R + \frac{1}{2}\left(M_\eta^2 + \frac{1}{6}R\right)^2 + \frac{2}{3}\Box M_A^2 + \frac{1}{6}\Box M_\eta^2\Big\} . \tag{4.15}$$

Finally, the bilinear form of the ghost action (3.10) can be written as

$$\hat{\mathcal{H}}_{gh} = \Box + M_{gh}^2\,, \quad \text{where} \quad M_{gh}^2 = m^2 + \frac{e^2}{\lambda}\,\xi\mathcal{K}\,. \tag{4.16}$$

The ghost contribution to the divergencies have the form

$$-i\,\mathrm{Tr}\ln\hat{\mathcal{H}}_{gh}\Big|_{\mathrm{div}} = \frac{1}{2(4\pi)^2(2-\omega)} \int d^4x\,\sqrt{-g}\,\mathrm{tr}\Big\{ -\frac{1}{90}\left(R^2_{\mu\nu\alpha\beta} - R^2_{\mu\nu} + \Box R\right) - \tag{4.17}$$
$$- \left(M_{gh}^2 + \frac{1}{6}\,R\right)^2 - \frac{1}{3}\Box\left(M_{gh}^2 + \frac{1}{6}\,R\right)\Big\} .$$

The total expression for the divergencies of the vacuum effective action in the theory with SSB is

$$
\begin{aligned}
\bar{\Gamma}_1^{(\mathrm{div})} &= \frac{1}{2(4\pi)^2(2-\omega)} \times \\
&\times \int d^4x\sqrt{-g}\Big\{\frac{1}{2}(3m^4+m_h^4) - \left(\xi-\frac{1}{6}\right)(m^2+m_h^2)R - \frac{2\,m^2}{3}\,R + \\
&\qquad + \left(\frac{3e^2m^2}{\lambda}+m^2+3m_h^2\right)\xi\mathcal{K} + \frac{7}{60}C^2_{\mu\nu\alpha\beta} - \frac{8}{45}\,E + \left(\xi-\frac{1}{6}\right)^2 R^2 - \\
&\qquad - \frac{7}{90}\Box R + \left(\frac{e^2}{2\lambda}+\frac{2}{3}\right)\xi\Box\mathcal{K} + \left(\frac{3e^4}{2\lambda^2}+\frac{e^2}{\lambda}+5\right)(\xi\mathcal{K})^2 - \\
&\qquad - \left[\left(\xi-\frac{1}{6}\right)\left(\frac{e^2}{\lambda}+4\right)+\frac{2e^2}{3\lambda}\right] R\cdot\xi\mathcal{K} - 2e^2(\nabla v)^2\Big\}\,,
\end{aligned}
\tag{4.18}
$$

where the last term can be integrated by parts and replaced by (neglecting the surface term)

$$
-\,2\,e^2(\nabla v)^2 \;\longrightarrow\; 2e^2v^2\,(\xi R - \xi\mathcal{K}) = m^2(\xi R-\xi\mathcal{K}) + \frac{e^2}{\lambda}\,\xi\mathcal{K}\,(\xi R-\xi\mathcal{K})\,. \tag{4.19}
$$

Finally, disregarding the surface terms, we have

$$
\begin{aligned}
\bar{\Gamma}_1^{(div)} &= \frac{1}{2(4\pi)^2(2-\omega)} \times \\
&\times \int d^4x\sqrt{-g}\left\{\frac{1}{2}(3m^4+m_h^4) - \left(\xi-\frac{1}{6}\right)m_h^2 R - \frac{1}{2}m^2R + \left(\frac{e^2}{\lambda}m^2+m_h^2\right)\times\right. \\
&\qquad \times 3\,\xi\mathcal{K} + \frac{7}{60}C^2_{\mu\nu\alpha\beta} - \frac{8}{45}E + \left(\xi-\frac{1}{6}\right)^2 R^2 + \left(\frac{3e^4}{2\lambda^2}+5\right)(\xi\mathcal{K})^2 - \\
&\qquad \left. - \left[4\left(\xi-\frac{1}{6}\right)+\frac{e^2}{2\lambda}\right] R\cdot\xi\mathcal{K}\right\},
\end{aligned}
\tag{4.20}
$$

The expression above differs from what is usually expected from the divergencies of the quantum field theory in an external gravitational field. Along with the usual local terms, there are many terms which look local only when they are expressed in terms of v or $\mathcal{K}$. After replacing the expressions (2.4) and (4.4) into (4.20), it becomes clear that these terms are indeed non-local with respect to the background metric $g_{\mu\nu}$.

Since the appearance of non-local divergences is quite a surprising result, let us explain it in more details. The $\xi\mathcal{K}$-dependent terms may be rewritten, using (4.4), in terms of VEV v as follows (here m_1^2 is some dimensional parameter)

$$
\begin{aligned}
m_1^2\xi\mathcal{K} &= 2\lambda m_1^2 v^2 - \mu_0^2 m_1^2\,, \\
R\,\xi\mathcal{K} &= 2\lambda R\,v^2 - R\mu_0^2\,, \\
(\xi\mathcal{K})^2 &= 4\lambda^2v^4 - 4\lambda v^2\mu^2 + \mu_0^4\,.
\end{aligned}
\tag{4.21}
$$

Thus, the following new structures emerge in the counterterms:

$$
v^4\,, \qquad m_1^2v^2\,, \qquad Rv^2\,. \tag{4.22}
$$

This can be compared to the expression for divergences for an ordinary real massive scalar field χ with quartic interaction and nonminimal coupling

$$\Gamma^{\rm div}_{\rm scal} = -\frac{\mu^{n-4}}{\varepsilon}\int d^n x\sqrt{-g}\left\{\frac{\lambda^2}{8}\chi^4 - \frac{\lambda}{2}\chi^2 m_1^2 - \frac{\lambda}{2}\chi^2(\xi-\frac{1}{6})R\right\} + (\text{vac. terms})\,, \qquad (4.23)$$

where χ is a background scalar field, independent on the metric. Of course, this is quite similar to (4.22). The only difference between the two cases is that the VEV v in (4.22) is not independent on metric but instead is given by the nonlocal expression (2.4).

The positive feature is that all necessary counterterms (local and non-local) have the form of the induced gravitational action (2.8) plus the standard local gravitational terms such as the cosmological constant, Einstein-Hilbert and higher derivative terms. The important consequence is that the theory with SSB is renormalizable in curved space-time. However, in order to achieve renormalizability, the corresponding non-local terms must be included into the classical action of vacuum along with the usual local ones [7, 8]

$$S_{\text{vac},1} = -\int d^4x\sqrt{-g}\,\left\{a_1\,C^2_{\mu\nu\alpha\beta} + a_2\,E + a_3\,\Box R + a_4\,R^2 + a_5\,R + a_6\right\}. \qquad (4.24)$$

Using the expression for the divergences (4.20), one can establish the necessary set of the non-local terms

$$S_{\text{vac},2} = -\int d^4x\sqrt{-g}\,\left\{q_1\,\xi\mathcal{K} + q_2\,R\xi\mathcal{K} + q_3\,(\xi\mathcal{K})^2\right\}. \qquad (4.25)$$

After these terms are included, we have the total classical action $S_{\text{vac},1}+S_{\text{vac},2}$, which must be compared to the induced action of vacuum in the theory with SSB (2.10). In the theory with the complete vacuum action $S_{\text{vac},1}+S_{\text{vac},2}$, the non-local counterterms can be removed by renormalizing the parameters q_1, q_2, q_3. The observable values of these parameters are defined as sums of the induced ones plus the values of the renormalized vacuum parameters — the last depend on the choice of the renormalization condition.

One may worry about the potential problems with the non-local terms in the classical action of vacuum (and also in the induced action of vacuum), such as unitarity of the S-matrix. Let us remember that what we obtained is a direct consequence of the SSB phenomena in curved space. The appearance of the non-localities looks inevitable in this framework. On the other hand, the non-localities which we are discussing here possess very special properties:

- They emerge in the action of an external gravitational field, therefore they have nothing to do with the unitarity of the S-matrix of the quantum theory. The very concept of an external field implies that it satisfies proper equations of motion, with certain boundary and initial conditions. Therefore, there is no real locality in the vacuum sector anyway.

•• The physical effect of the non-localities is extremely weak, at least in the finite order in the curvature expansion. The point is that the non-localities are related to the Green functions $(\Box + 4\lambda v_0^2)^{-1}$ and therefore their effect may be observed only at the distances comparable to $(\sqrt{\lambda} v_0)^{-1}$. Since, according to the experimental data $\sqrt{\lambda} v_0 > 50\,\text{GeV}$ in the case of the SM of particle physics, the non-local terms are irrelevant in the modern Universe.

- ••• The conclusion of the previous point may be not completely safe. One can foresee the following two exceptions: i) The resummation of the curvature expansion series may produce the massless-type non-localities. As far as we do not have control over these series,[4] we can not exclude this possibility, which may lead to very interesting cosmological consequences like the IR running of the CC due to the remnant quantum effects of the decoupled heavy particles [10]. ii) If we consider a very light scalar with the SSB (e.g. some version of a quintessence), then the non-localities may become potentially observable and in particular may lead to the slight modification of the Newton law. Such modifications may either put restriction on the corresponding model or become relevant for the Dark Matter problem. This possibility is an interesting problem which deserves special investigation.

Using the expression for the divergences (4.20), we can derive the β-functions for the parameters of the usual vacuum action (4.24) and for the parameters of the non-local action (4.25) which are necessary in the theory with the SSB. Let us write down the $\overline{MS}$-scheme β-functions for those parameters of the vacuum action which do not correspond to the surface terms. In the expressions below we used the mass m_h (because this mass appears in the non-localities at the classical level (2.10)) and the relation $m^2 = (e^2/2\lambda) \cdot m_h^2$.

$$\beta_1^{\overline{MS}} = \frac{7}{60(4\pi)^2}\,, \qquad \beta_2^{\overline{MS}} = -\,\frac{8}{45(4\pi)^2}\,, \qquad \beta_4^{\overline{MS}} = \frac{1}{(4\pi)^2}\left(\xi - \frac{1}{6}\right)^2, \tag{4.26}$$

$$\beta_5^{\overline{MS}} = -\,\frac{m_h^2}{(4\pi)^2}\left(\frac{e^2}{4\lambda} + \xi - \frac{1}{6}\right), \qquad \beta_6^{\overline{MS}} = \frac{m_h^4}{(4\pi)^2}\left(\frac{3e^2}{2\lambda} + 1\right), \tag{4.27}$$

$$\beta_{q1}^{\overline{MS}} = \frac{3\,m_h^2}{(4\pi)^2}\left(\frac{e^4}{2\lambda^2} + 1\right), \qquad \beta_{q2}^{\overline{MS}} = -\,\frac{1}{(4\pi)^2}\left[4\left(\xi - \frac{1}{6}\right) + \frac{e^2}{2\lambda}\right],$$

$$\beta_{q3}^{\overline{MS}} = \frac{1}{(4\pi)^2}\left(\frac{3e^4}{2\lambda^2} + 5\right). \tag{4.28}$$

Let us remark that the signs of the β-functions $\beta_1^{\overline{MS}}$, $\beta_2^{\overline{MS}}$ for the higher derivative terms in this paper are different from the ones of [1] due to the opposite sign of the classical action. In the next section we shall derive the physical β-functions in the mass-dependent (momentum subtraction) renormalization scheme and will use the above expressions (4.27) and (4.28) in order to check these β-functions in the UV limit.

5. Renormalization and decoupling in the SSB theory

In order to observe the decoupling of the massive degrees of freedom at low energies in the theory with SSB, one has to apply the mass-dependent scheme of renormalization in curved space-time. The most economic way of doing this is to perform the covariant calculation

[4] These series are obtained by inserting expansion (2.4) into (2.8) plus the quantum corrections.

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using the heat-kernel method. The existing results for the heat kernel enable one to perform practical calculations at the second order in curvature [20, 21]. This method of calculation is completely equivalent to the derivation of the polarization operator of graviton (or vertices, in the higher orders in curvatures) due to the quantum effects of the matter fields [1].

The calculation of the effective action can be mainly performed using the results for the massive vector and massive scalar.[5] The one-loop contribution to the euclidean effective action is given by the sum of three terms

$$\bar{\Gamma}^{(1)} = -\frac{1}{2}\,\mathrm{Tr}\ln\hat{\mathcal{H}}_h - \frac{1}{2}\,\mathrm{Tr}\ln\hat{\mathcal{H}} + \mathrm{Tr}\ln\hat{\mathcal{H}}_{gh}\,. \tag{5.1}$$

The operators $\hat{\mathcal{H}}_{gh}$ and $\hat{\mathcal{H}}$ correspond to the fields with the mass m, while the operator $\hat{\mathcal{H}}_h$ correspond to the field with the mass m_h. Therefore we can use, for each of these the three operators, the standard Schwinger-DeWitt representation, e.g.

$$-\frac{1}{2}\,\mathrm{Tr}\ln\hat{\mathcal{H}} = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\,e^{-sm^2}\;\mathrm{tr}\;K(s)\,. \tag{5.2}$$

An explicit expression for the heat kernel $K(s)$ of the operator $\hat{1}\Box + \hat{\Pi}$ has been found in [20, 21], and we can use this result directly for the three operators of interest.

Since the practical calculations has been described in the second reference of [1], we will not go into details here. Let us present the final expression for the $\mathcal{O}(R^2)$-terms in the effective action

$$\begin{aligned}
\bar{\Gamma}^{(1)} = \frac{1}{2(4\pi)^2}\int d^4x\sqrt{-g}\Big\{ & \frac{3m^4+m_h^4}{2}\cdot\left(\frac{1}{2-w}+\frac{3}{2}\right)+\frac{3m^4}{2}\ln\left(\frac{4\pi\mu^2}{m^2}\right)+\\
&+\frac{m_h^4}{2}\ln\left(\frac{4\pi\mu^2}{m_h^2}\right)+\left[\left(\frac{3e^2}{\lambda}+1\right)\xi\mathcal{K}-\left(\xi+\frac{1}{2}\right)R\right]\cdot m^2\times\\
&\times\left[\frac{1}{2-w}+\ln\left(\frac{4\pi\mu^2}{m^2}\right)+1\right]+\left[3\xi\mathcal{K}-\left(\xi-\frac{1}{6}\right)R\right]\cdot m_h^2\times\\
&\times\left[\frac{1}{2-w}+\ln\left(\frac{4\pi\mu^2}{m_h^2}\right)+1\right]+\frac{1}{2}C_{\mu\nu\alpha\beta}\times\\
&\times\left[\frac{7}{30(2-w)}+\frac{13}{60}\ln\left(\frac{4\pi\mu^2}{m^2}\right)+\frac{1}{60}\ln\left(\frac{4\pi\mu^2}{m_h^2}\right)+k_W^{\mathrm{total}}(a,a_h)\right]\times\\
&\times C^{\mu\nu\alpha\beta}+R\left[\left(\xi-\frac{1}{6}\right)^2\cdot\left(\frac{1}{2-w}+\ln\left(\frac{16\pi^2\mu^4}{m^2m_h^2}\right)\right)+\right.\\
&\qquad\left.+k_R(a)+k_R(a_h)+k_R^{gv}(a)\right]R+\\
&+R\left[\left(1+\frac{3e^2}{\lambda}\right)\cdot\frac{3Aa^2-a^2-12A}{18\,a^2}+\frac{3A_ha_h^2-a_h^2-12A_h}{6\,a_h^2}-\right.\\
&\quad-3\left(\xi-\frac{1}{6}\right)\cdot\left(\frac{1}{2-w}+\ln\left(\frac{4\pi\mu^2}{m_h^2}\right)+2A_h\right)-\\
&\quad-\left(\left(1+\frac{e^2}{\lambda}\right)\left(\xi-\frac{1}{6}\right)+\frac{2e^2}{3\lambda}\right)\cdot
\end{aligned}$$

[5] The only piece which requires a special calculation is the non-diagonal sector of the operator $\hat{\Pi}$ in (4.14).

$$\cdot \left(\frac{1}{2-w} + \ln \left(\frac{4\pi\mu^2}{m_h^2} \right) + 2A \right) \Big] \xi \mathcal{K} +$$

$$+ \xi\mathcal{K} \Big[\left(\frac{3e^4}{2\lambda^2} + \frac{e^2}{\lambda} + \frac{1}{2} \right) \cdot \left(\frac{1}{2-w} + \ln \left(\frac{4\pi\mu^2}{m^2} \right) + 2A \right) +$$

$$+ \frac{9}{2} \left(\frac{1}{2-w} + \ln \left(\frac{4\pi\mu^2}{m_h^2} \right) + 2A_h \right) \Big] \xi\mathcal{K} -$$

$$- 2e^2 (\nabla_\mu v) \left[\frac{1}{2-w} + \ln \left(\frac{4\pi\mu^2}{m^2} \right) + 2A \right] (\nabla^\mu v) \Big\} , \tag{5.3}$$

where

$$A = A(a) = 1 - \frac{1}{a} \ln \frac{1 + a/2}{1 - a/2} \qquad \text{and} \qquad a^2 = a^2(m) = \frac{4\Box}{\Box - 4m^2} . \tag{5.4}$$

Of course, $A_h = A(a_h)$ and $a_h^2 = a^2(m_h)$. In the expression (5.3) we used the following notations for the formfactors:

$$k_W^{\text{total}}(a, a_h) = \frac{8A_h}{15a_h^4} + \frac{2}{45a_h^2} + A + \frac{8A}{5a^4} - \frac{8A}{3a^2} + \frac{2}{15a^2} - \frac{88}{450} , \tag{5.5}$$

$$k_R(a) = A\Big(\xi - \frac{1}{6}\Big)^2 - \frac{A}{6}\left(\xi - \frac{1}{6}\right) + \frac{2A}{3a^2}\left(\xi - \frac{1}{6}\right) + \frac{A}{9a^4} - \frac{A}{18a^2} + \frac{A}{144} + $$
$$+ \frac{1}{108\, a^2} - \frac{7}{2160} + \frac{1}{18}\left(\xi - \frac{1}{6}\right) , \tag{5.6}$$

$$k_R^{gv}(a) = \frac{13}{1080} - \frac{A}{24} + \frac{1}{54a^2} + \frac{2A}{9a^4} + \frac{A}{9a^2} \tag{5.7}$$

(this corresponds to a massive vector with an extra compensating scalar [1]).

The divergent part of the one-loop part of the effective action (5.3) is exactly the eq. (4.18). Concerning the finite part, it is easy to see that the non-localities of the expression (5.3) have two sources. First of all we meet the tree-level non-localities inside each $\xi\mathcal{K}$, and moreover there are a-dependent and A-dependent non-localities which have the structure similar to the one for the usual massive fields [1]. If considering the β-functions, the first type of the non-localities does not matter, hence one can expect to meet the same result as in the $\overline{MS}$-scheme in the high energy regime and the standard decoupling [1] at low energies.

It is not difficult to confirm the last statement by direct calculation. Using the formfactors of the expression (5.3) we can derive the physical β-functions. For this end one has to perform the subtraction at the euclidean momentum square $-\Box \to p^2 = M^2$ and then apply the receipt

$$\beta_C = \lim_{n \to 4} M \frac{dC}{dM} \tag{5.8}$$

for the effective charge C. The coincidence with the $\overline{MS}$-scheme β-function in the UV provides an efficient verification of the calculations.

For the usual parameters of the vacuum action, corresponding to the local terms, we obtain the following results:

1. The physical β-functions for the classical cosmological constant a_6, inverse Newton constant a_5 and for the coefficient q_1 of the non-local term $\xi\mathcal{K}$ are not visible in this framework, for the reasons which were already explained above and

in [1]. Unfortunately, at this point there is no qualitative difference between the theory were the masses are introduced from the very beginning and the theory with SSB.

2. For the coefficient of the $C^2_{\mu\nu\alpha\beta}$ term we obtain, after some algebra

$$\beta_1 = -\frac{1}{(4\pi)^2}\left[\frac{17}{90} - \frac{1}{6a^2} - \frac{a^2}{16} + \frac{(a^2-4)(a^4-8a^2+8)\,A}{16a^4} + \frac{3A_h(a_h^2-4)-a_h^2}{18\,a_h^4}\right], \tag{5.9}$$

that is the general result for the one-loop β-function, valid at any scale. In the high energy UV limit $p^2 \gg m_h^2$ we obtain

$$\beta_1^{UV} = \frac{7}{60\,(4\pi)^2} + \mathcal{O}\left(\frac{m_h^2}{p^2}\right). \tag{5.10}$$

that agrees with the $\overline{\text{MS}}$-scheme result (4.26). In the IR limit $p^2 \ll m^2$ we meet

$$\beta_1^{IR} = \frac{3}{112\,(4\pi)^2}\left(\frac{2\lambda}{e^2} + \frac{1}{45}\right)\cdot\frac{p^2}{m_h^2} + \mathcal{O}\left(\frac{p^4}{m_h^4}\right). \tag{5.11}$$

We have found that the IR limit of the β_1 in the theory with SSB demonstrates the decoupling, similar to the simple massive theory [1]. There is a weak dependence on the parameter λ/e^2 in the IR, but the very fact that the decoupling occurs in the gravitational vacuum sector of the theory with SSB does not depend on the magnitude of the scalar coupling λ.

3. The overall β_4-function has the following form:

$$\begin{aligned}\beta_4 = -\frac{1}{8\,(4\pi)^2}\Bigg[&\frac{(4-a^2)}{144\,a^4}\,(5\,a^4\,A - 20\,a^2 - 5\,a^4 - 240\,A - 24\,a^2\,A) + \\ &+ \frac{(4-a^2)\,(a^2\,A - a^2 - 12\,A)}{6\,a^2}\left(\xi - \frac{1}{6}\right) + \\ &+ \frac{\left(4-{a_h}^2\right)\left({a_h}^2\,A_h - a_h^2 - 12\,A_h\right)}{6\,{a_h}^2}\left(\xi - \frac{1}{6}\right) \\ &+ \left({a_h}^2\,A_h - {a_h}^2 - 4\,A_h + a^2\,A - a^2 - 4\,A\right)\cdot\left(\xi - \frac{1}{6}\right)^2\Bigg].\end{aligned} \tag{5.12}$$

Here we did not separate the terms proportional to $(\xi - 1/6)$ into β_3, as we did in the previous publication [1]. Indeed, such distinction can be done in case it is necessary, then we shall have an independent β_3-function. But, at the moment, our main interest is the interface between the UV and IR limits in the Effective Action and there is no need to enter into these details.

In the UV limit $a \to 2$, $a_h \to 2$ we meet

$$\beta_4^{UV} = \frac{1}{(4\pi)^2}\left(\xi - \frac{1}{6}\right)^2, \tag{5.13}$$

that is exactly $\beta_4^{\overline{MS}}$ from the eq. (4.26).

Indeed, we are most interested in the IR limit $a \to 0$, $a_h \to 0$ of the β_4-function

$$\beta_4^{IR} = \frac{1}{(4\pi)^2}\Big[\frac{11\,\lambda}{3780\,e^2} + \frac{1}{180}\Big(1+\frac{2\lambda}{e^2}\Big)\cdot\Big(\xi-\frac{1}{6}\Big) - \\ - \frac{1}{12}\Big(1+\frac{2\lambda}{e^2}\Big)\cdot\Big(\xi-\frac{1}{6}\Big)^2\Big]\,\frac{p^2}{m_h^2} + \mathcal{O}\Big(\frac{p^4}{m_h^4}\Big)\,, \tag{5.14}$$

demonstrating the standard quadratic form of decoupling, similar to the β_1 case.

4. Let us now consider the nontrivial new β-functions for the parameters of the non-local terms (4.25). The β_{q2}-function has the form

$$\beta_{q2} = \frac{1}{4\,(4\pi)^2}\Big[\big(a^2 A - a^2 - 4A + 3a_h^2 A_h - 3a_h^2 - 16A_h\big)\cdot\Big(\xi - \frac{1}{6}\Big) + \\ + \frac{a^2(A-1)(3e^2-\lambda)}{12\,\lambda} - \frac{4A(3\,e^2+\lambda)}{a^2\lambda} + \frac{(2A-1)e^2}{\lambda} + \frac{(4A-1)}{3} + \\ + \frac{(a_h^2-4)(A_h a_h^2 - a_h^2 - 12A_h)}{4a_h^2}\Big]. \tag{5.15}$$

The UV limit shows perfect correspondence with the $\overline{MS}$-scheme expression (4.28)

$$\beta_{q2}^{UV} = -\,\frac{1}{(4\pi)^2}\,\Big[\frac{e^2}{2\,\lambda} + 4\Big(\xi-\frac{1}{6}\Big)\Big]\,, \tag{5.16}$$

while in the IR limit we meet usual decoupling

$$\beta_{q2}^{IR} = -\,\frac{1}{(4\pi)^2}\,\Big[\frac{\lambda}{90\,e^2} - \frac{7}{60} - \Big(\frac{1}{2}+\frac{\lambda}{3e^2}\Big)\Big(\xi-\frac{1}{6}\Big)\Big]\,\frac{p^2}{m_h^2} + \mathcal{O}\Big(\frac{p^4}{m_h^4}\Big)\,. \tag{5.17}$$

5. The β_{q3}-function has the form

$$\beta_{q3} = \frac{1}{8\,(4\pi)^2}\,\Big[9\,\big({a_h}^2 + 4\,A_h - {a_h}^2\,A_h\big) + \Big(1+\frac{3\,e^4}{\lambda^2}\Big)\,\big(a^2 + 4\,A - a^2\,A\big)\Big]\,. \tag{5.18}$$

In the UV limit we meet correspondence with the $\overline{MS}$-scheme β_{q3}-function (4.28)

$$\beta_{q3}^{UV} = \frac{1}{(4\pi)^2}\,\Big(5+\frac{3\,e^4}{2\,\lambda^2}\Big)\,, \tag{5.19}$$

while in the IR limit there is usual decoupling

$$\beta_{q3}^{IR} = \frac{1}{(4\pi)^2}\,\Big(\frac{\lambda}{6\,e^2} + \frac{3}{4} + \frac{e^2}{\lambda}\Big)\,\frac{p^2}{m_h^2} + \mathcal{O}\Big(\frac{p^4}{m_h^4}\Big)\,. \tag{5.20}$$

Thus, the IR behaviour of the new vacuum parameters q_2 and q_3 in the higher derivative sector of the theory is very similar to the one for the "old" parameters a_1 and a_4. In all cases we meet soft quadratic decoupling when the energy-momentum parameter of the linearized gravity p^2 becomes much smaller than the masses of the particles induced by SSB.

6. Conclusions

We have considered the quantum fields theory with SSB in an external gravitational field. The SSB produces non-local terms in the induced action of vacuum already at the classical level — the phenomenon which was not, up to our knowledge, described before in the literature. The non-local terms emerge due to the coordinate dependence of the curvature scalar and do not show up in the spaces of constant curvature, or in the theory with the minimal coupling between scalar and curvature. The appearance of the non-localities does not break such important properties of the quantum fields theory in an external gravitational field, as unitarity and renormalizability. The unitarity is preserved in the matter sector, because the non-local terms emerge only in the action of external gravitational field. From the formal point of view, in the theory with SSB one has to include the non-local terms into the classical action of vacuum in order to provide the renormalizability of the theory.

The physical effects of the new non-local terms do not look very important, at least in the framework of the well-established physical theories. The reason is that the non-localities enter the action through the insertion of the Green functions corresponding to the scalar particle with the mass $m_h = 2\sqrt{\lambda}v_0$, where v_0 is the VEV for the flat space theory. In the case of the Standard Model, this mass has the order of magnitude around 100 GeV, and of course the effect of the non-localities becomes significant only at very small distances. Therefore, the effect of these terms may be important only in the earliest periods in the history of the universe. Moreover, in order to achieve these small distances one needs to use very high energies. Then the symmetry should be restored because the temperature of the radiation interacting with the quantum fields is always much greater than the energy of the gravitational quantas. As far as the symmetry gets restored, the induced non-local terms do not show up.

The situation may be quite different for the very light fields, like e.g. quintessence (one of candidates for the role of a time-dependent Dark Energy). The quintessence is supposed to be an extremely light field and therefore, if its mass is due to the SSB, it should produce the non-local effects which may be observable.

The most important result of our work is that the decoupling really takes place for the theories with SSB in curved space-time. We have investigated the renormalization of vacuum parameters in the theory with SSB and found that, in the low-energy limit, they all vanish quadratically, in accordance with the Appelquist and Carazzone theorem. In this respect, the vacuum sector of the theory with SSB is different from the matter sector, (see, e.g. [11]) where the decoupling does not take place.

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